Simplyfing Rational Expressions

Rational Expression: a ratio of two polynomials
Domain of a Rational Expression: all real numbers except those that make the denominator equal to zero

ex/ $\frac{x+2}{x+1}$ domain: $x \neq -1$

How to Simplify:
1) Factor both the numerator and denominator
2) Reduce the fraction by canceling out common factors
3) List restricted values (from

ex/

$$\frac{7x-28}{x^2-16} = \frac{7(x-4)}{(x+4)(x-4)} = \frac{7}{x+4} \quad x \neq \pm 4$$

$$\frac{2x^2+11x+5}{3x^2+17x+10} = \frac{(2x+1)(x+5)}{(3x+2)(x+5)} = \frac{2x+1}{3x+2} \quad x \neq -2/3, -5$$

$$\frac{6x^2-19x+3}{4x^2-36} = \frac{(6x-1)(x-3)}{4(x-3)(x+3)} = \frac{6x-1}{4(x+3)} \quad x \neq 3, -3$$

$$\frac{x-8}{x^2-2x-48} \cdot \frac{4x^2+40x}{x+10} = \frac{x-8}{(x-8)(x+6)} \cdot \frac{4x(x+10)}{x+10} = \frac{4x}{x+6}; \quad x \neq 8, -6, -10$$

$$\frac{2b^2-12b}{b+5} \div \frac{b-6}{b+5} = \frac{2b(b-6)}{b+5} \cdot \frac{b+5}{b-6} = 2b; \quad x \neq -5, 6$$

$x \neq 5$ $\quad$ $x \neq 6$

$$\frac{1}{x+1} + \frac{x}{x-6} - \frac{5x-2}{x^2-5x-6}$$

$$= \frac{1}{x+1} + \frac{x}{x-6} - \frac{5x-2}{(x-6)(x+1)}$$

$$= \left(\frac{1}{x+1}\right)\left(\frac{x-6}{x-6}\right) + \left(\frac{x}{x-6}\right)\left(\frac{x+1}{x+1}\right) - \frac{5x-2}{(x-6)(x+1)}$$

$$= \frac{x-6}{(x+1)(x-6)} + \frac{x^2+x}{(x+1)(x-6)} - \frac{5x-2}{(x+1)(x-6)}$$

$$= \frac{(x-6)+(x^2+x)-(5x-2)}{(x+1)(x-6)}$$

$$= \frac{x-6+x^2+x-5x+2}{(x+1)(x-6)}$$

$$= \frac{x^2-3x-4}{(x+1)(x-6)} = \frac{(x+1)(x-4)}{(x+1)(x-6)} = \frac{x-4}{x-6}; \quad x \neq -1, 6$$

Simplyfing Rational Expressions Practice

1) $\frac{2x^2+5x-3}{6x^2+18x}$

2) $\frac{6x^2-19x+3}{4x^2-36}$

3) $\frac{10x^2y}{15x^5y^2}$

4) $\frac{3x^2-3x}{3x^3-6x^2+3x}$

5) $\frac{\frac{1}{x+1}+\frac{3}{x-3}}{\frac{2}{x-3}-\frac{1}{x+1}}$

Answer Key

1) $\frac{2x^2+5x-3}{6x^2+18x} = \frac{(2x-1)(x+3)}{6x(x+3)} = \frac{2x-1}{6x}$; $x \neq 0, -3$

2) $\frac{6x^2-19x+3}{4x^2-36} = \frac{(6x-1)(x-3)}{4(x-3)(x+3)} = \frac{6x-1}{4(x+3)}$; $x \neq \pm 3$

3) $\frac{10x^2y}{15x^5y^2} = \frac{2 \cdot 5 \cdot x \cdot x \cdot y}{3 \cdot 5 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y} = \frac{2}{3x^3y}$

4) $\frac{3x^2-3x}{3x^3-6x^2+3x} = \frac{3x(x-1)}{3x(x^2-2x+1)} = \frac{x-1}{(x-1)(x-1)} = \frac{1}{x-1}$; $x \neq 0, 1$

5) $\frac{\frac{1}{x+1}+\frac{3}{x-3}}{\frac{2}{x-3}-\frac{1}{x+1}} = \frac{\left(\frac{1}{x+1}+\frac{3}{x-3}\right)(x+1)(x-3)}{\left(\frac{2}{x-3}-\frac{1}{x+1}\right)(x+1)(x-3)}$

$= \frac{\frac{1(x+1)(x-3)}{x+1} + \frac{3(x+1)(x-3)}{x-3}}{\frac{2(x+1)(x-3)}{x-3} - \frac{1(x+1)(x-3)}{(x+1)}}$

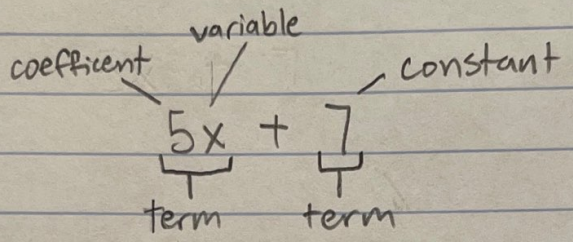
$= \frac{(x-3)+3(x+1)}{2(x+1)-1(x-3)} = \frac{x-3+3x+3}{2x+2-x+3} = \frac{4x-2}{x+5}$

Simplifying Algebraic Expressions

Term: a single mathematical expression, separated by +/- (ex/ 9, 5x, $2n^2$)

Constant: a term that is just a number and does not contain a variable (ex/ 3, 8)

Coefficent: a number multiplied by a variable (ex/ $\boxed{3}y$, $\boxed{7}x^3$)



Like Terms: two or more terms that have the exact same variable and exponents, the coefficents do not have to be the same (ex/ $\boxed{x^2} + \underline{7x} - \textcircled{2} + \underline{9x} - \textcircled{3} + \boxed{4x^2}$)

How to Simplify

1) Use the distributive property to get rid of grouping symbols, like brackets or parentheses
2) Combine like terms by addition or subtraction, including constants (because all constants are like terms)

ex/

$4a + 2 + a$

$4a + a + 2$

$5a + 2$

$5(2y-3) - 8$

$10y - 15 - 8$

$10y - 23$

$7 - (2x+1) + 9(4-x) + 10$

$7 - 2x - 1 + 36 - 9x + 10$

$-2x - 9x + 7 - 1 + 36 + 10$

$-11x + 52$

$$\left(\frac{x^{-1}yz^{-2}}{y^{-5}zx^3}\right)^{-1} = \frac{y^{-5}zx^3}{x^{-1}yz^{-2}} = \frac{zx^3x^1z^2}{y \cdot y^5} = \frac{x^4z^3}{y^6}$$

$(-9m - 8n) + (-2mn - 2n^2 + m^2) + (-n^2 + 4mn)$

$-9m - 8n + 2mn - 2n^2 + m^2 - n^2 + 4mn$

$m^2 - 2n^2 - n^2 - 2mn + 4mn - 9m - 8n$

$m^2 - 3n^2 + 2mn - 9m - 8n$

Simplifying Algebraic Expression Practice

1) $5(2y-3)-8$

2) $-7(x-1)+5(2x+3)$

3) $5(2x^3+5x^2-8)-3(4x^3+5x-2)$

4) $7x^2+3y^2+2y^2-4x^2$

5) $-(x-2)+x(6x-7)$

6) $(2x+y)^2+(2y^2+x^2+4x)-(-y+x)-5xy$

7) $4r(r+6)-2r^2(3r-1)$

8) $\left(\frac{a^2b^{-1}}{ab}\right)\left(\frac{a^3}{b^{-4}}\right)^{-3}$

Answer Key

1) $5(2y-3)-8$

$10y-15-8 = 10y-23$

2) $-7(x-1)+5(2x+3)$

$-7x+7+10x+15$

$3x+22$

3) $5(2x^3+5x^2+8)-3(4x^2+5x-2)$

$10x^3+25x^2+40-12x^2-15x+6$

$10x^3+13x^2-15x-34$

4) $7x^2+3y^2+2y^2-4x^2$

$(7x^2-4x^2)+(3y^2+2y^2)$

$3x^2+5y^2$

5) $-(x-2)+x(6x-7)$

$-x+2+6x^2-7x$

$6x^2-8x+2$

6) $(2x+y)^2+(2y^2+x^2+4x)-(-y+x)-5xy$

$(2x+y)(2x+y)+2y^2+x^2+4x+y-x-5xy$

$4x^2+4xy+y^2+2y^2+x^2+4x+y-x-5xy$

$5x^2+3y^2+3x+y-xy$

7) $4r(r+6)-2r^2(3r-1)$

$4r^2+24r-6r^3+2r^2$

$-6r^3+6r^2+24r$

$-6r(r^2-r-4)$

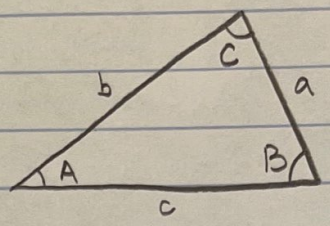
8) $\left(\frac{a^2b^{-1}}{ab}\right)\left(\frac{a^3}{b^{-4}}\right)^{-3} = \left(\frac{a^2b^{-1}}{ab}\right)\left(\frac{b^{-4}}{a^3}\right)^3$

$= \left(\frac{a^2b^{-1}}{ab}\right)\left(\frac{b^{-12}}{a^9}\right) = \frac{a^2}{ab\cdot b^1\cdot b^{12}\cdot a^9}$

$= \frac{a^{\cancel{2}1}}{a^{\frac{10}{8}}b^{14}} = \frac{1}{a^8b^{14}}$

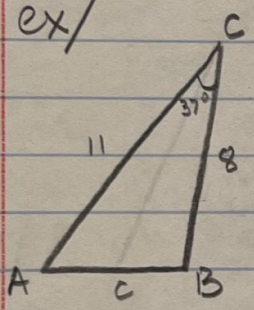
Law of Cosines (Cosine Rule)

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$
$$b^2 = a^2 + c^2 - 2ac\cos(B)$$
$$a^2 = b^2 + c^2 - 2bc\cos(A)$$



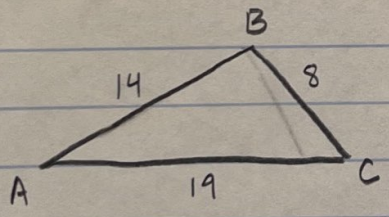
- used to solve an oblique triangle when either:
  - the lengths of two sides and the measure of the included angle is known (SAS) or
  - the lengths of the three sides are known (SSS)

ex/



$\angle C = 37°$
$a = 8$
$b = 11$

$c^2 = a^2 + b^2 - 2ab\cos(C)$
$c^2 = 8^2 + 11^2 - 2(8 \cdot 11)\cos(37)$
$c^2 = 64 + 121 - 176(0.798)$
$c^2 = 44.44$
$c = \sqrt{44.44} \approx 6.67$



$b^2 = a^2 + c^2 - 2ac\cos(B)$
$\cos(B) = \frac{b^2 - a^2 - c^2}{-2ac} = \frac{19^2 - 8^2 - 14^2}{-2(8)(14)} \approx -0.4508$
$B = \cos^{-1}(-0.45089) \approx 116.80°$

To find the other two angles, it is simplest to use the Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{8}{\sin A} \approx \frac{19}{\sin 116.80°} \approx \frac{14}{\sin C}$$

$$\sin A \approx \frac{8\sin 116.80°}{19}$$

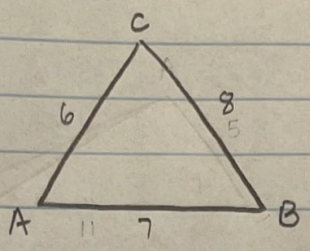
$$\angle A \approx 22.08°$$

$$\sin C \approx \frac{14\sin 116.80°}{19}$$

$$\angle C \approx 41.12°$$

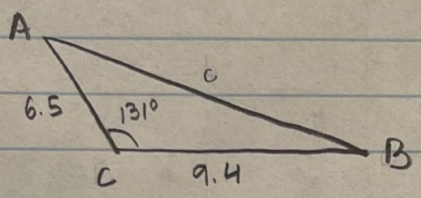
Law of Cosines Practice

1)



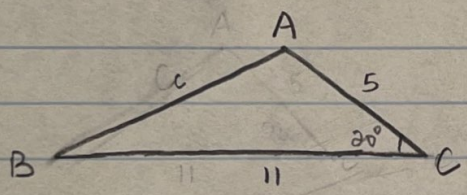
Find the angle measurement for ∠C

2)



Find the length of side c

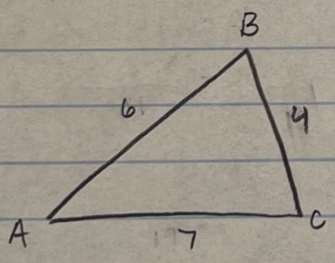
3)



Find the length of side c

Challenge: Find the measure of the remaining angles

4)



Find the measurement for angle C

Answer Key

1) $c^2 = a^2 + b^2 - 2ab\cos(C)$

$\cos(C) = \frac{a^2+b^2-c^2}{2ab}$

$= \frac{8^2+6^2-7^2}{2\times 8\times 6} = \frac{64+36-49}{96} = \frac{51}{96} = 0.53125$

$C = \cos^{-1}(0.53125)$

$C = 57.9°$

2) $c^2 = a^2 + b^2 - 2ab\cos(C)$

$c^2 = 9.4^2 + 6.5^2 - (2\cdot 9.4\cdot 6.5)\cos(131°)$

$c^2 = 210.76$

$c = \sqrt{210.76}$

$c = 14.5°$

3) $c^2 = a^2 + b^2 - 2ab\cos C$

$c = \sqrt{a^2+b^2-2ab\cos C}$

$= \sqrt{11^2+5^2-2(11)(5)(\cos 20°)}$

≈ 6.52

$\sin A = \frac{11\sin 20°}{6.53}$

$A \approx 144.82°$

$\sin B = \frac{5\sin 20°}{6.53}$

$B \approx 15.2°$

4) $\cos B = \frac{b^2-a^2-c^2}{-2ac} = \frac{7^2-4^2-6^2}{-2(4)(6)} \approx 0.0625$

$B = \cos^{-1}(0.0625) \approx 86.42°$